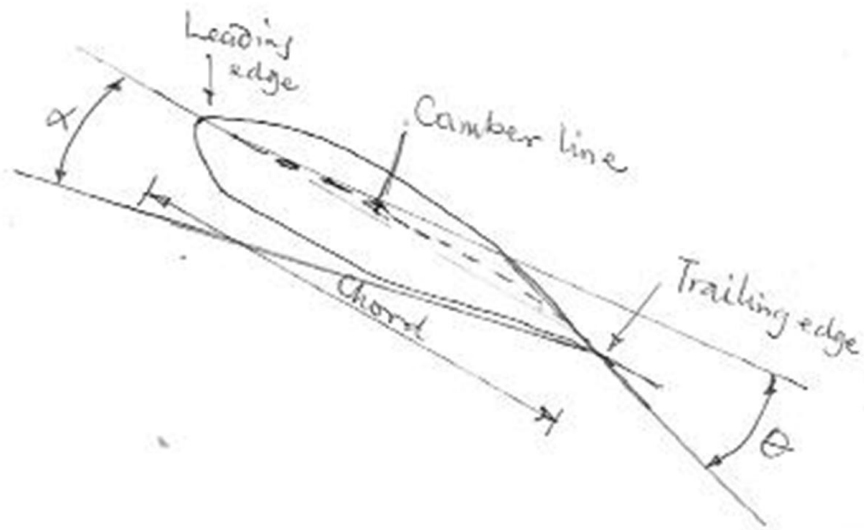
Flow Past An Infinitely Long Aerofoil.

An aerofoil may be defined as a streamlined body design to produce lift. There are other lift-producing surfaces such as hydrofoils or circular arcs.



Consider the aerofoil or airfoil sketched above, the leading edge is the front or upstream edge facing the direction of flow, while the trailing edge is the rear, or downstream edge. Other important terms relating to airfoil are as follows:

- **Chord line:** This is a straight line joining the centres of curvature of the leading and trailing edges.
- **Chord, C:** The length of chord line between the leading and trailing edges.
- **Camber Line:** The centerline of the airfoil section
- **Camber,** δ **:** The maximum distance between the camber line and the chord line.
- **Deviation** θ **:** The angle between the tangent to camber line at trailing edge and the tangent to camber line at leading edge.
- **Angle of attack** (incidence)**:** The angle between the direction of the relative motion and the chord line
- **Pressure coefficient Cp: (** $p - p_0 / \frac{1}{2}\rho U_o$ **)** where ρ is the local pressure and P_0 is the pressure far upstream of the aerofoil where velocity is Vo.

The primary purpose of an aerofoil is to produce lift when placed in a fluid stream. It will of course, experience drag at the same time. In order to minimize drag, an aerofoil is a streamlined body. A measure of its usefulness as a wing section of an aircraft or as a blade section for a pump or turbine is the ratio of lift to drag. The higher this ratio is, the better the aerofoil, in the sense that it is capable of producing high lift at a small drag penalty. In an aircraft it is the lift on the wing surface which maintains the plane in the air. At the same time, it is the drag which absorbs all the engine power necessary for the craft's forward motion.

The lift/drag ratio

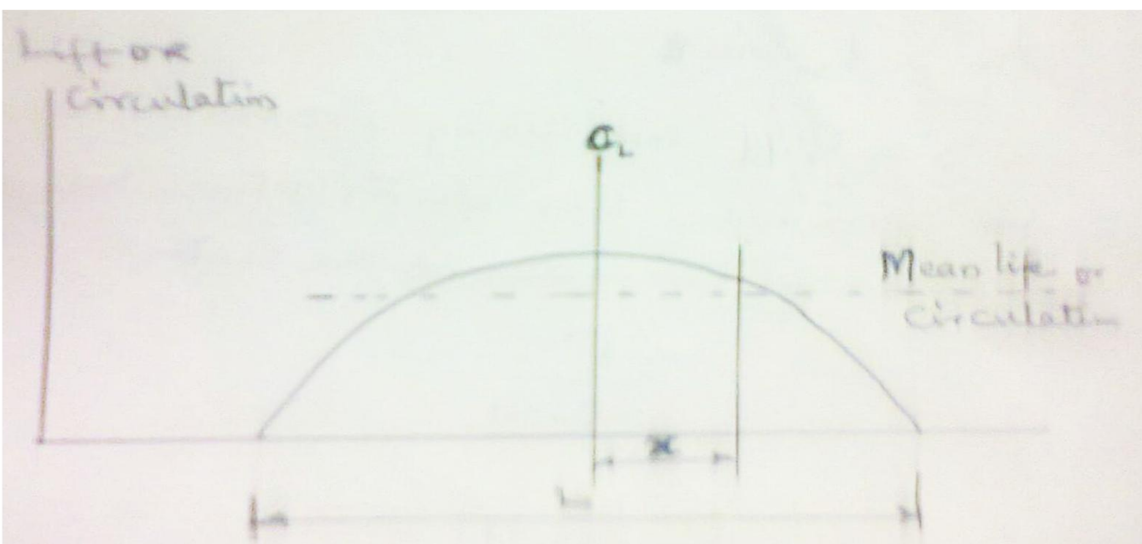
$$\frac{Lift}{Drag} = \frac{\frac{1}{2}\ell C_L U_o^2 A}{\frac{1}{2}\ell C_D U_o^2 A} = \frac{C_L}{C_D}$$

Flow Past an Aerofoil of Finite Length

When an aerofoil is subjected to lift force, the pressure on its underside is greater than that on the top. This pressure difference between the upper and the lower surface causes flow around the tips of the aerofoil from the underside to the upper surface. This end flow affects the rest of the flow pattern in the following manner. The flow on the underside is deflected towards the tips of the aerofoil in order to supply the necessary end flow, whereas the flow at the top of the aerofoil is deflected from the tips towards the centre.

Since there is end flow at the tips, the pressure difference between the top and bottom surfaces of an aerofoil must decrease from a maximum at the middle towards the tips where it is zero.

Consequently, the circulation around the aerofoil finite span must also decrease from its maximum value Γ_a at the centerline towards zero at the tips. The distribution is to be approximated to an ellipse as shown below



Distribution of lift along a wing's span

A further consequence of the tip vortices is that they induce a downward velocity component which is known as downwash velocity $\overline{vi}$. Its presence means that the relative velocity of motion between the fluid and the aerofoil is no longer the free stream velocity Uo but velocity U, deflected from Uo by an angle $\in$ known as the induced angle of incidence. The resulting geometry is shown below. What follows is that, in accordance with the definition of lift, which stipulates that it is perpendicular to the relative direction of motion, the true lift is normal to U. However, since it is more convenient and customary to relate lift and drag to the direction of the free stream relative to the aerofoil, the true lift L_o is resolved into L, the component perpendicular to U_o, and D_i, the component parallel to U_o. This latter component, which is in the same direction as drag is known as induced drag and is added to pressure drag and the skin friction drag to give the total drag on an aerofoil. The expression for induced drag is derived as follows. The true lift per unit length of span is given by

$$L_o = L_o U \tau;$$

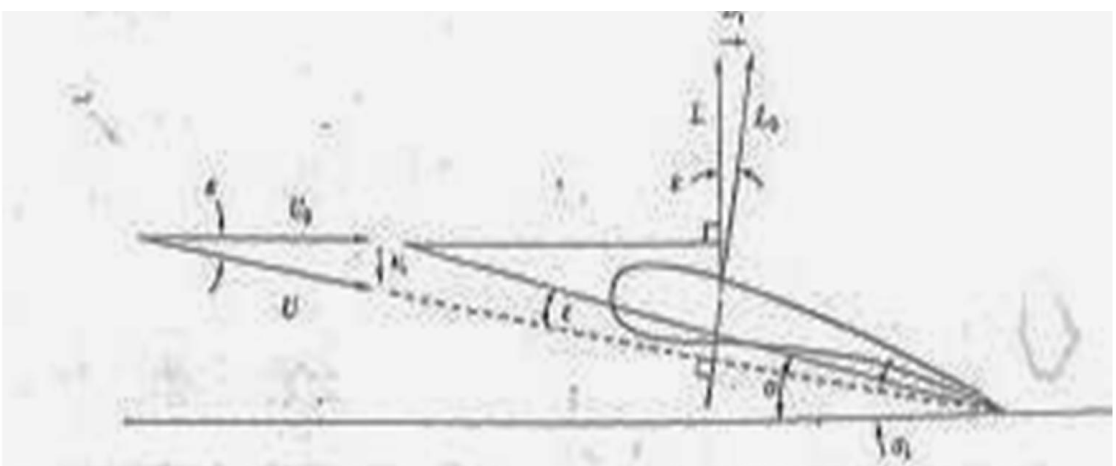
Hence, the induced drag per unit span

$$D_i = L_o Sin \in$$

But $\in = V_i / U$ and, using Prandtl's approximation for elliptical spanwise lift distribution that $V_i = \dfrac{\Gamma_o}{2bU}$, where

and

$$\begin{aligned} D_i &= \ell U \Gamma \left(\Gamma_o / 2bU \right) \\ &= \ell \Gamma \left(\Gamma_o \big/ 2b \right) \end{aligned}$$



Induced drag

Now, for the elliptic spanwise distribution of Γ ,

$$\Gamma = \Gamma_o \left[1 - \left(2x/b\right)^2\right]^{1/2}$$

Where x is the distance from the centerline. Thus, the induced drag for the total span,

$$D_1 = \frac{\ell}{2b}\Gamma_o^2 \int_{-b/2}^{+b/2} \left[1 - \left(2x/b\right)^2\right]^{1/2} dx = \frac{\ell}{2b}\Gamma_o^2 \frac{b\Pi}{4} \qquad (vi)$$

$$= \ell\Pi\Gamma_o^2 / 8$$

is obtained by substitution $2x/b = Sin\,\theta$ But

$$L_0 = \int_{+b/2}^{+b/2} \ell U\Gamma dx = \ell U\Gamma_{+b/2}^{+b/2}\left[1 - \left(2x/b\right)^2\right]^{1/2} dx$$

$$= \ell U\Gamma_o b\frac{\Pi}{4}$$

From which

$$\Gamma_o = 4L_o / \ell Ub\Pi$$

And, substituting into eqn. (vi)

$$D_i = (\ell\Pi/8)\,(4L_o / \ell Ub\Pi)^2 = 2L_o^2 / \ell\Pi U^2 b^2$$

However, from similar triangles

$$L_0/U = L/U_0$$

and, hence,

$$D_i = \left(2/\ell\Pi b^2\right)\left(L/U_o\right)^2$$

If the coefficient of induced drag is defined as

$$C_{Di} = \frac{D_i}{\frac{1}{2}\ell U^2 A}$$

And since $C_L = \dfrac{L}{\frac{1}{2}\ell U^2 A}$, , by substitution

$$C_{Di} = \frac{D_i}{L/C_L} = \frac{C_L 2L^2}{L\ell\Pi b^2 U^2} = 2C_L \frac{L}{\ell\Pi b^2 U^2}$$

$$= 2C_L \frac{\frac{1}{2}C_L\ell U^2 A}{\ell\Pi b^2 U^2} = C_L^2 \frac{A}{\Pi b^2} = C_L^2 \frac{cb}{\Pi b^2}$$

$$= \frac{C_L^2}{\Pi} \times \frac{c}{b}$$

But

$$\frac{c}{b} = \frac{1}{Aspect\ Ratio},$$

So that

$$C_{Di} = \frac{C_L^2}{\Pi\ (Aspect\ \ Ratio)}$$

This equation shows that a large aspect ratio minimizes the induced drag, as would be expected.

Example 1

A wing of an aircraft of 10m span and 2m mean chord is designed to develop a lift of 45kN at a speed of 400km/h. A 1/20 scale model of the wing section is tested in a wind tunnel at 500m/s and ρ = 5.33 kg/m^3. The total drag measured is 400N. Assuming that the wing tunnel data refer to a section of infinite span, calculate the total drag for the full-size wing. Assume an elliptical lift distribution and take air density as 1.2kg/m^3.

Solution

Wing area,

$$A = 2 \text{ x } 10 = 20\text{m}^2$$

Coefficient of drag from the model data,

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 A} = \frac{\mathbf{400}}{\frac{1}{2} X\ 5.3\ X\ 500^2\ X\ ^{20}/_{20^2}} = \mathbf{0.012.}$$

For the prototype,

$$U = 400 \text{ km/h} = 111.1\text{m/s}$$

And the lift coefficient,

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 A} = \frac{\mathbf{45000}}{\frac{1}{2} \boldsymbol{X} \mathbf{1.2} \boldsymbol{X} \mathbf{111.1^2} \boldsymbol{X} \mathbf{20}} = \mathbf{0.304}$$

Now , assuming an elliptical distribution, the coefficient of induced drag,

$$C_{D_0} = C_L^2/\pi(AR) = (0.304)^2/\pi\left(\frac{10}{2}\right) = 0.0059.$$

Hence, the total drag coefficient,

$$C_{D_i} = C_D + C_{D_0} = 0.012 + 0.0059 = 0.0179$$

And the total drag on the wing,

$$D = \frac{1}{2} C_{D_i} \rho U^2 A = \frac{1}{2} \text{ x } 0.0179 \text{ x } 1.2\ (111.1)^2 \text{ x } 20 = 2648.9 \text{ N}$$

Therefore,

$$D = 2.65 \text{ kN}.$$